Basic Rules of Logic

The symbols P, Q, and R denote arbitrary statements, and **T** and **F** denote "True" and "False" respectively. The following are fundamental logical equivalences.

Name	**Equivalence**
Exclusive Middle	$P \vee (\neg P) \equiv \mathbf{T}$ $P \wedge (\neg P) \equiv \mathbf{F}$
Double Negation Law	$\neg(\neg P) \equiv P$
Idempotence Laws	$P \vee P \equiv P$ $P \wedge P \equiv P$
Identity Laws	$P \wedge \mathbf{T} \equiv P$ $P \vee \mathbf{F} \equiv P$
Domination Laws	$P \vee \mathbf{T} \equiv \mathbf{T}$ $P \wedge \mathbf{F} \equiv \mathbf{F}$
Commutative Laws	$P \vee Q \equiv Q \vee P$ $P \wedge Q \equiv Q \wedge P$
Associative Laws	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
Distributive Laws	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
De Morgan's Laws	$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$ $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$
Material Implication	$P \to Q \equiv (\neg P) \vee Q$
Contraposition	$P \to Q \equiv (\neg Q) \to (\neg P)$
Biconditional Expansion	$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$

The following are fundamental logical deduction rules.

Name	**Inference Rule**
Modus Ponens	$P \to Q,\ P \quad \therefore Q$
Modus Tollens	$P \to Q,\ \neg Q \quad \therefore \neg P$
Disjunctive Syllogism	$P \vee Q,\ \neg P \quad \therefore Q$
Hypothetical Syllogism	$P \to Q,\ Q \to R \quad \therefore P \to R$
Dilemma	$P \vee Q,\ P \to R,\ Q \to R \quad \therefore R$
Reductio Ad Absurdum	$(\neg P) \to [Q \wedge (\neg Q)] \quad \therefore P$
Conjunctive Simplification	$P \wedge Q \quad \therefore P$
Conjunctive Addition	$P,\ Q \quad \therefore P \wedge Q$
Disjunctive Addition	$P \quad \therefore P \vee Q$

1. Use truth tables to verify each logical equivalence and each inference rule above.
2. Use truth tables to verify each equivalence below.

   (a) $P \vee Q \equiv (\neg P) \to Q$
   (b) $P \wedge Q \equiv \neg(P \to (\neg Q))$
   (c) $\neg(P \to Q) \equiv P \wedge (\neg Q)$
   (d) $(P \to Q) \wedge (P \to R) \equiv P \to (Q \wedge R)$
   (e) $(P \to R) \wedge (Q \to R) \equiv (P \vee Q) \to R$
   (f) $(P \to Q) \vee (P \to R) \equiv P \to (Q \vee R)$
   (g) $(P \to R) \vee (Q \to R) \equiv (P \wedge Q) \to R$
   (h) $P \leftrightarrow Q \equiv (P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$
   (i) $P \leftrightarrow Q \equiv (\neg P) \leftrightarrow (\neg Q)$
   (j) $\neg(P \leftrightarrow Q) \equiv P \leftrightarrow (\neg Q)$

3. Use a string of logical equivalences to verify each equivalence claimed in question 2.
4. Use disjunctive normal forms to verify each equivalence claimed in question 2.